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A new mechanism for a naturally small Dirac neutrino mass

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A mechanism is proposed in which a right-handed neutrino zero mode and a right-handed charged lepton zero mode can be localized at the same place along an extra compact dimension while having markedly different spreads in their wave functions: a relatively narrow one for the neutrino and a rather broad one for the charged lepton. In their overlaps with the wave function for the left-handed zero modes, this mechanism could produce a natural large hierarchy in the effective Yukawa couplings in four dimensions, and hence a large disparity in masses.

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The possibility that a neutrino has a small non-vanishing mass (of order of a few electronvolts or less) is by now universally accepted and backed by various neutrino oscillation experiments [1]. However what is not established at this moment is the nature of the mass itself. Is it of the Dirac type or is it of the Majorana type? There are ongoing and planned experiments designed to address this very issue [2, 3]. Despite the general belief that the masses of the neutrinos are of the Majorana type, it is prudent to keep in mind that they could be of the pure Dirac type and to explore scenarios pertaining to this latter possibility. The final words must evidently come from experiments.

The common prejudice in favor of a Majorana mass is a very natural one. The see-saw mechanism [4] with a ratio of a small scale (electroweak) /a large scale (Grand Unified scale, etc..) provides a plausible explanation for the smallness of the Majorana neutrino mass. If one simply enlarges the minimal SM by endowing it with right-handed neutrinos, the neutrino masses would naturally be of the Dirac type and $m_\nu/m_l = g_{Y\nu}/g_{Yl} \lesssim 10^{-9}$. It is then considered to be highly unnatural (fine tuning) to put in by hand $g_{Y\nu} \lesssim O(10^{-11})$ if $g_{Yl} \sim 10^{-2}$ (roughly the value of the Yukawa coupling of the heaviest charged lepton). It would be more natural if $g_{Y\nu} \lesssim O(10^{-11})$ were to arise dynamically. This was the motivation of Ref. [5] where Dirac neutrino masses arise at the one-loop level and can be "naturally" small.

In this paper, another approach concerning a naturally small Dirac mass for the neutrinos is presented. A very interesting connection to the quark sector will be shown. It makes use of the interesting notion that the effective Yukawa coupling (in four dimensions) controlling in part the value of the fermion mass is proportional to the size of the wave function overlap between left-handed and right-handed fermions along the compact fifth (spatial) dimension.

Let us start with the case where there is one extra spatial dimension- denoted by y- compactified on an orbifold S_1/Z_2 and having length L. This has been shown to contain chiral zero modes in four dimensions [6], a desirable feature in the construction of the SM. These chiral zero modes (for either left or right-handed mode) can be written as $\psi^0_{L,R}(x,y) = \psi_{L,R}(x)\xi_{L,R}(y)$, where $x \equiv x^\mu$. For free fields, $\xi_{L,R}(y) =$ constant and the wave function is uniformly spread over the extra dimension. When a background scalar field with a kink solution is introduced and is made to interact with the fermion, it was found that the zero mode is localized at the center of the domain wall (the zero of the kink) and rapidly vanishes outside it. As it has been discussed in Ref. [7], one can arrange domain walls at various locations along y so as to place fermions at different points.

What has not been discussed so far is the possibility that each fermion interacts with more than one background scalar field and the ensuing implications on the shape and location of the wave function along y. This turns out to be very relevant to the issue of naturally small Dirac neutrino masses, as well as to the splitting between the up and down sectors of the quarks.

Let us first start with the simple case of one fermion, ψ, interacting with two background scalar fields, ϕ_1 and ϕ_2, as follows

$$\mathcal{L}_Y = f_1 \bar{\psi}\psi\,\phi_1 + f_2 \bar{\psi}\psi\,\phi_2\,. \qquad (1)$$

For simplicity, let us take the non-vanishing chiral zero mode to be a right-handed one. This can be accomplished by choosing the appropriate Z_2 parity. (The other chirality can be treated in the same way.) As in Ref. [6, 7], kink solutions will be assumed for ϕ_1 and ϕ_2, which will be denoted by $h_1(y)$ and $h_2(y)$ and are given by

$$h_{1,2}(y) = v_{1,2}\tanh\{\mu_{1,2}\,(y - y_{1,2})\}\,, \qquad (2)$$

where $\mu_{1,2} = (\lambda_{1,2}/2)^{1/2} v_{1,2}$ ($\lambda_{1,2}$ being the coefficients of the quartic couplings), and where we have allowed for the locations of the zeros of the kinks to be at $y_{1,2}$. The equation for ξ_R is given by

$$\partial_y \xi_R(y) + (f_1 h_1(y) + f_2 h_2(y))\xi_R(y) = 0\,, \qquad (3)$$

The normalized zero mode wave function for the fermion is now given by

$$\xi_R(y) = k \exp(-\int_0^y dy' (f_1 h_1(y') + f_2 h_2(y'))) , \quad (4)$$

where k is a normalization factor so that $\int_0^L \xi_R^2 = 1$. To illustrate the above discussion, let us, for simplicity, set $y_1 = y_2 = 0$ and call $f_{1,2}/(\lambda_{1,2}/2)^{1/2} = C_{1,2}$. It can easily be seen that $\xi_R(y)$ now takes the simple form

$$\xi_R(y) = k e^{-C_1 \ln(\cosh(\mu_1 y)) - C_2 \ln(\cosh(\mu_2 y))} . \quad (5)$$

To illustrate (5) graphically, we shall simply put $C_1 = \pm 1$, $C_2 = \pm 1$, with the appropriate normalization factor k and, in an arbitrary unit of masses, $\mu_1 = 1$, $\mu_2 = 0.9$. (A more general case can be straightforwardly carried out.) Since $1/\mu$ corresponds to the thickness of the domain wall, these illustrative numerical values correspond to two domain walls of comparable thicknesses. (However, this is *not* "fine tuning".) In Fig. 1, we show two cases (the sum and the difference respectively): (a) $C_1 = 1$, $C_2 = 1$; (b) $C_1 = 1$, $C_2 = -1$. These two cases correspond to two different wave functions which are denoted by WFa and WFb respectively. Also shown are the two kinks located at the same point but endowed with slightly different profiles. We also show another fermion wave function (e.g. a left-handed one), denoted by WFL which is localized separately at a point far enough from the two domain walls. The domain wall with which it interacts is chosen to be of comparable size to the other two.

From Fig. 1, one can see that WFa is markedly different from WFb. It is much narrower while the latter is rather broad. (The difference in the maxima of WFa and WFb reflects the two different normalization factors which are computed numerically here.) For illustrative purposes, the (left-handed) wave function, WFL, on the left of Fig. 1 is chosen to have a tiny overlap with WFa. However, its overlap with WFb is large, due to the spread of WFb. In the parlance of effective couplings in four dimensions, WFL-WFa and WFL-WFb overlaps would give rise to two markedly different effective couplings: a tiny one and a much larger one respectively. If these were the Yukawa couplings responsible for the masses, one would have obtained two widely separated masses. One could envision the possibility that the (right-handed) fermion involved in the WFL-WFa overlap is the right-handed neutrino and the one involved in the WFL-WFb overlap is the (right-handed) charged lepton. But how does one realize this scenario in a realistic model?

Although we have alluded to a scenario where the aforementioned fermions are leptons, one can also imagine a similar one for the quarks, with a notable difference: The split of the overall mass scales between the up and down sector is about a factor of 60 or so. In this case, one might imagine moving WFL closer to the center of WFa and WFb, and reverse the coupling of the right-handed quarks (coupling to the sum for the down quark (WFa) and to the difference for the up quark (WFb), all to the same background scalar fields). Fig. 2 illustrates this scenario. By choosing the appropriate location for WFL, one can reproduce the above desired ratio. It is intriguing that, with the *same* two background scalars, the ratio of the overall mass scales between the up and down sectors for *both* leptons and quarks can be obtained by choosing the appropriate *relative* (to the right-handed fermions) distance for WFL(lepton) and WFL(quark). (Only these relative distances are relevant here.) For simplicity, the "origin" in Fig. 1 and 2 really means the relative location of the right-handed fermions and not their actual positions along y.

To summarize, given two background scalar fields and two different fermions, the coupling of one fermion to the sum of the scalars (Case (a)) gives rise to a "narrow" localized wave function while the one which couples to the difference (Case (b)) gives rise to a "broad" localized wave function. Similar conclusions can be reached for other values of μ_1 and μ_2 as long as they are not too different from one another.

Notice that a fermion in five dimensions is a Dirac fermion which can be written as $\psi = (\psi_L + \psi_R)$, where $\psi_{L,R} = P_{L,R}\psi$, with $P_{L,R} = (1 \mp \gamma_5)/2$ being the usual four-dimensional chiral projection operator. The Z_2 symmetry in conjunction with the boundary conditions will select one of the two chiralities to have a non-vanishing zero mode, depending on the chosen Z_2 parity. In addition to the SM, let us introduce an extra symmetry $SU(2)_R$ which could either be global or gauged as in the L-R model [8]. The notations used for the lepton doublets in four dimensions are as follows: $l_L = (\nu_L, e_L)$ ($SU(2)_L$ doublet) and $l_R = (\nu_R, e_R)$ ($SU(2)_R$ doublet), where ν and e are generic names for the neutral and charged leptons. Let us now embed these leptons in five-dimensional spinors, namely $L^{\{L\}}(x,y) = (l_L^{\{L\}} + l_R^{\{L\}})$ and $L^{\{R\}}(x,y) = (l_L^{\{R\}} + l_R^{\{R\}})$. Here, $l_L(x)$ is the zero mode of $l_L^{\{L\}}(x,y)$, and $l_R(x)$ is the zero mode of $l_R^{\{R\}}(x,y)$. In the absence of interactions, these zero modes depend only on x and are not localized along y.

Let us now concentrate on $L^{\{R\}}$. Let us introduce two background scalar fields: $\Phi_T = \vec{\phi}_T . \frac{\vec{\tau}}{2}$ and ϕ_S, which are a triplet and a singlet under $SU(2)_R$ respectively, and hence the subscripts T and S. Furthermore, as usual, these background scalars are assumed to transform under Z_2 as $\phi_S(x,y) \to -\phi_S(x, L-y)$, $\Phi_T(x,y) \to -\Phi_T(x, L-y)$ as in Ref. [6]. The Yukawa coupling (in five dimensions) can now be written as

$$\mathcal{L}_{Y2} = f_T^{(l)} \bar{L}^{\{R\}} \Phi_T \, L^{\{R\}} + f_S^{(l)} \bar{L}^{\{R\}} \phi_S \, L^{\{R\}} . \quad (6)$$

With Φ_T being an $SU(2)_R$ triplet, its minimum energy configuration can be written as $\langle \Phi_T \rangle = \langle \phi_T^3 \rangle \tau_3/2$, and explicitely as

$$\langle \Phi_T \rangle = \begin{pmatrix} h_T(y) & 0 \\ 0 & -h_T(y) \end{pmatrix} \quad . \tag{7}$$

For ϕ_S, one has $\langle \phi_S \rangle = h_S(y)$. Let us write the zero mode for $l_R^{\{R\}}(x,y)$ as $l_R^{0,\{R\}}(x,y) = l_R(x)\xi_R(y)$. The equation governing the behaviour of ξ_R is now

$$\partial_y \xi_R(y) + (f_T^{(l)} \langle \Phi_T \rangle + f_S^{(l)} \langle \phi_S \rangle)\xi_R(y) = 0\,. \tag{8}$$

With $\xi_R = (\xi_R^\nu, \xi_R^e)$, (8) splits into

$$\partial_y \xi_R^\nu(y) + (f_S^{(l)} h_S(y) + f_T^{(l)} h_T(y))\xi_R^\nu(y) = 0\,, \tag{9}$$

$$\partial_y \xi_R^e(y) + (f_S^{(l)} h_S(y) - f_T^{(l)} h_T(y))\xi_R^e(y) = 0\,, \tag{10}$$

The solutions are

$$\xi_R^{\nu,e}(y) = k_{\nu,e} \exp(-\int_0^y dy' (f_S^{(l)} h_S(y') \pm f_T^{(l)} h_T(y'))\,, \tag{11}$$

where k_ν and k_e are normalization factors and where $f_{S,T}$ are assumed to be positive. Eq. (11) has the same form as Eq. (4). The analysis is identical to the generic case discussed above when one makes identifications of various coefficients, namely $1,2 \to S,T$ and $k \to k_{\nu,e}$. As for the quark sector, one would like the right-handed Up wave function to be "broad" and the right-handed Down wave function to be "narrow". This can be accomplished by writing the following term $-f_T^{(q)} \bar{Q}^{\{R\}}\, \Phi_T\, Q^{\{R\}} + f_S^{(q)} \bar{Q}^{\{R\}}\, \phi_S\, Q^{\{R\}}$, with $f_{T,S}^{(q)}$ assumed to be positive. With this, $\xi_U(y)$ will behave like $\xi_e(y)$, and $\xi_D(y)$ like $\xi_\nu(y)$. Eqs. (11) is seen to be an explicit realization of the phenomenon presented in this paper.

In the following discussion, we will ignore fermion mixings and concentrate on the overall mass scales, which usually mean the heaviest fermion for each sector. A Yukawa interaction responsible for the masses would be of the form $g_{Y,l} \bar{L}^{\{L\}}\, H\, L^{\{R\}} + h.c.$, where H is a Higgs field, and similarly for the quarks. If $SU(2)_R$ were a global symmetry then H would be a 2×2 matrix of the form $H = (\phi, \tilde{\phi})$ with $\tilde{\phi} = i\sigma_2 \phi^*$. Assuming an even Z_2-parity for H so that it has a non-trivial zero mode $H^{(0)}$, it follows that $\langle H^{(0)} \rangle = diag(v, -v)/\sqrt{2}$ with $v = 246$ GeV. If $SU(2)_R$ were a gauge symmetry as in the LR model then $\langle H^{(0)} \rangle = diag(v_1, -v_2)$ with $v_1^2 + v_2^2 = 246$ GeV. For simplicity, we will first concentrate on the global $SU(2)_R$ case and see what we can learn from there. In this case, one only has to deal with one fundamental scale v. If we denote by ξ_L the corresponding wave function for the left-handed zero mode, the effective Yukawa couplings in four dimensions are

$$g_{eff,\nu} = g_{Y,l} \int_0^L dy \xi_L^l \xi_R^\nu \,;\, g_{eff,e} = g_{Y,l} \int_0^L dy \xi_L^l \xi_R^e\,, \tag{12}$$

and similarly for the quark couplings, $g_{eff,U}$ and $g_{eff,D}$. The overall mass scales will then be: $g_{eff,\nu}\, v/\sqrt{2}$ and $g_{eff,e}\, v/\sqrt{2}$ for the neutrino and charged lepton sectors respectively, and $g_{eff,U}\, v/\sqrt{2}$ and $g_{eff,D}\, v/\sqrt{2}$ for the Up and Down quark sectors respectively. In this scenario, the ratio of the two mass scales for the leptons and for the quarks are simply ratios of the overlaps. The fundamental Yukawa couplings $g_{Y,l,q}$ will come in when one wants to set the absolute scales of the masses.

Although our original motivation was to find a natural reason for a small Dirac neutrino mass, one can turn the argument around and apply the proposed mechanism to the quark sector first followed by a "prediction" for the lepton sector. Basically, the main philosophy of [7] is that the effective Yukawa couplings g_{eff} can be small even if the *fundamental* Yukawa couplings g_Y were of O(1). Let us assume here- for the sake of discussion- that all fundamental Yukawa couplings $g_{Y,l,q} \gtrsim 0.1$. (There is no deep reason for this lower bound other than a generous definition of what O(1) means.) Let us illustrate our scenario with the parameters chosen for Figs. (1,2). (A general discussion is beyond the scope of this paper.) From Fig. 2, the "broad" wave function represents U_R and the narrow one for D_R. Since $g_{Y,q} v/\sqrt{2}$ is common, m_t/m_b is just the ratio of overlaps and is about 60. The actual value for $m_t(m_t) \sim 166$ GeV yields $g_{Y,q} \sim 2.5$. (Remember that, in four dimensions, it is $g_{eff,U}$ which is used in perturbation theory and, in this particular case, is about unity.) It is the same two kinks which interact with the leptons, albeit at a different location. We then move to Fig. 1, where the origin now refers to the location of the right-handed doublet. We can then move the left-handed doublet (WFL) to the left *until* the fundamental coupling $g_{Y,l} \sim 0.1$ (from m_τ). There, $m_\nu/m_\tau \sim 10^{-7}$. The neutrino mass is *very small*, albeit one with the presently-believed wrong value (~ 160 eV). Moving futher to the left until $m_\nu/m_\tau \sim 2 \times 10^{-9}$, one gets $g_{Y,l} \sim 0.16$ and $m_\nu \sim 4.4$ eV. It is amusing to push further to the left until $g_{Y,l} \sim 1$ giving $m_\nu/m_\tau \sim 2 \times 10^{-16}$ and $m_\nu \sim 4 \times 10^{-7}$ eV. This last example shows that the neutrino mass is automatically very small when $g_{Y,l} \sim g_{Y,q}$ and when one fixes the overlap $\int_0^L dy \xi_L^l \xi_R^e$ so that the charged lepton acquires a mass of the right magnitude.

From the above examples, one cannot fail but notice a deep connection between quark and lepton masses. The ratios of masses for each sector are basically ratios of effective Yukawa couplings, namely $R_L \equiv g_{eff,\nu}/g_{eff,l}$ and $R_Q \equiv g_{eff,D}/g_{eff,U}$. The only difference between these ratios is the distances between WFL and WFa,b for the quark (d_q) and lepton (d_l) sectors, and they might come from a deeper theory unifying quarks and leptons. What this might mean is that, if such a unification exists, one might be able to relate these two distances so that using e.g. d_q to calculate R_Q, one can deduce R_l since d_l and d_q will then be related. What does the above numerical example tell us? The mechanism which splits the right-handed doublet into a "broad" and a "narrow" wave functions fixes the ratio of the Up and Down masses for the quarks as well as for the leptons and naturally gives rise to a tiny Dirac neutrino mass if one requires that the fundamental Yukawa couplings (in the higher dimensions) are of O(1).

This paper is concentrated mainly on a mechanism for the disparity of the overall mass scales, and has not discussed the issue of mixings. This will be carried out in a separate publication for the lepton sector. The mass matrices for the quark sector were investigated in [9].

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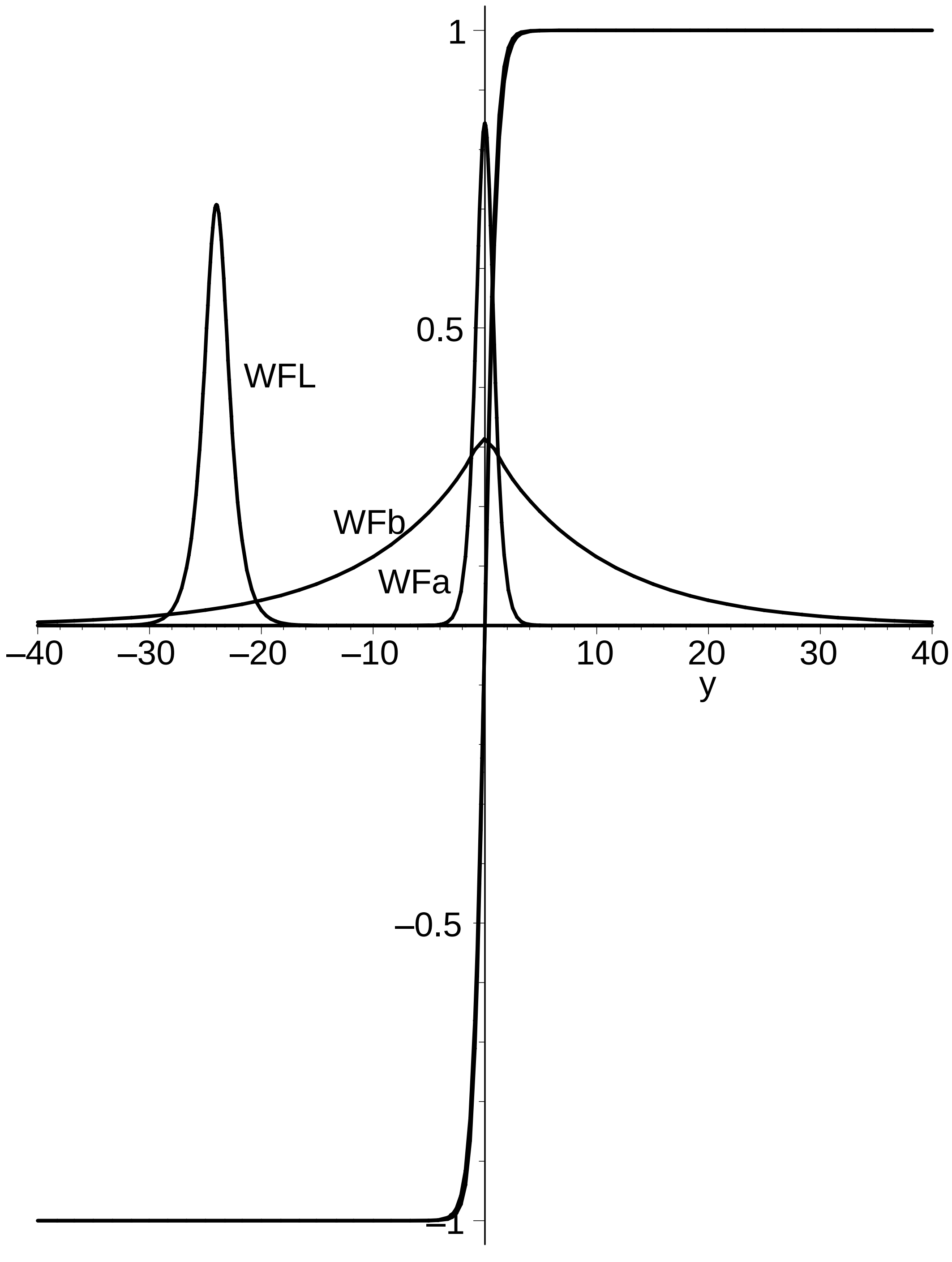

FIG. 1: The wave function (WF) overlap for the lepton case. WFa: right-handed neutrino WF. WFb: right-handed charged lepton WF. WFL: WF of left-handed lepton doublet. The two kinks are practically on top of one another.

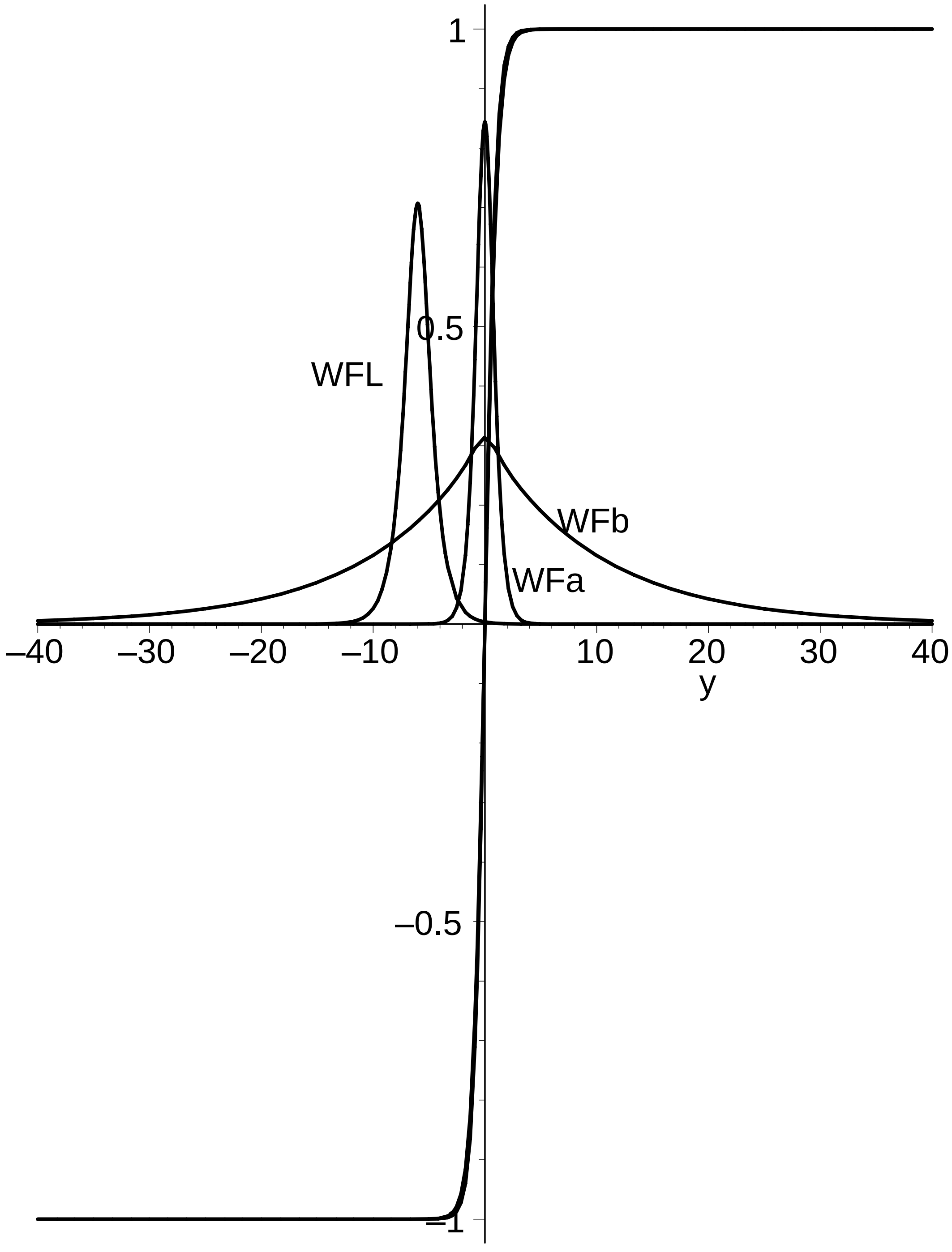

FIG. 2: The wave function (WF) overlap for the quark case. WFa: right-handed down quark WF. WFb: right-handed up quark WF. WFL: WF of left-handed quark doublet. The two kinks are practically on top of one another.